Chương 1

MÔ HÌNH PHÂN TÍCH SỐ LIỆU MẢNG

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1.1 Ôn tập về Kinh tế lượng

$$Y = f(X_2, X_3, ..., X_k) + \varepsilon = \beta_1 + \beta_2 X_2 + \beta_3 X_3 + ... + \beta_k X_k + u \quad (PRM)$$

$$f(X_2, X_3, ..., X_k) = \beta_1 + \beta_2 X_2 + \beta_3 X_3 + ... + \beta_k X_k \quad (PRF)$$

Giả thiết 1. Giá trị trung bình của sai số ngẫu nhiên (SSNN) bằng 0, nghĩa là: $\mathbf{E(u/X = x_i) = 0}$

Giả thiết 2. Phương sai của các SSNN bằng nhau, nghĩa là:

$\mathbf{Var(u/X = x_i) = \sigma^2}$

Giả thiết 3. Các SSNN không tương quan với nhau, $\mathbf{Cov(u_i , u_j) = 0}$

Giả thiết 4. Các SSNN và biến giải thích (X_i) không tương quan với nhau, nghĩa là: $\mathbf{Cov(u_i , X_i) = 0}$

Giả thiết 5. Các biến giải thích (X_2, …, X_k) độc lập tuyến tính

1.2 Một số khái niệm

- Số liệu chéo (Cross section data) là số liệu được thu thập tại một thời điểm cụ thể trên nhiều phần tử khác nhau.
- Số liệu chuỗi thời gian (Time series data) là số liệu được thu thập trên một phần tử nhưng tại nhiều thời điểm (thời gian) khác nhau.
- Số liệu mảng (Panel data) là sự kết hợp của số liệu chéo và số liệu chuỗi thời gian gồm **số liệu mảng cân bằng và số liệu mảng không cân bằng.**

Ví dụ 1.1 Xem file "Data_Ch1.xls"

1.3 Cấu trúc của panel data (Structure of Panel Data)

Mỗi biến (X) quan sát theo dữ liệu bảng được cấu tạo bởi 3 thành phần chính gồm:

- Biến X đang quan sát trên phần tử i (i = 1, 2, 3, …,n)
- Biến X đang quan sát ở thời gian t (t =1, 2, 3,…, T)
- Số lượng các biến (X_v) đang quan sát (v= 1, 2, …k)

Time unit (t)	Cross section unit (i)					
	1	2	3	…	N	$\bar{X}_{.t}$
1	X_{11}	X_{21}	X_{31}	…	X_{N1}	$\bar{X}_{.1}$
2	X_{12}	X_{22}	X_{32}	…	X_{N2}	$\bar{X}_{.2}$
3	X_{13}	X_{23}	X_{33}	…	X_{N3}	$\bar{X}_{.3}$
…	…	…	…	…	…	…
T	X_{1T}	X_{2T}	X_{3T}	…	X_{NT}	$\bar{X}_{.T}$
$\bar{X}_{i.}$	$\bar{X}_{1.}$	$\bar{X}_{2.}$	$\bar{X}_{3.}$	…	$\bar{X}_{N.}$	$\bar{X}$

Tổng quát: Một biến quan sát (X) theo cấu trúc của dữ liệu bảng sẽ có dạng là X_{it} (i =1, 2, ... n; t= 1, 2, 3,…,T) với

$$\overline{X_{i..}} = \frac{\sum_{t=1}^{T} X_{it}}{T}; \overline{X_{.t.}} = \frac{\sum_{i=1}^{N} X_{it}}{N}; \overline{X} = \frac{\sum_{i=1}^{N}\sum_{t=1}^{T} X_{it}}{NT};$$

1.4 Lợi ích trong phân tích số liệu mảng

- Số liệu mảng không có tính nhất quán giữa các số liệu được khảo sát. Các kỹ thuật ước lượng trong dữ liệu bảng sẽ đưa ra nhiều tính toán cho phù hợp.
- Số liệu mảng chứa nhiều thông tin hữu ích hơn, tính biến thiên nhiều hơn, ít hiện tượng đa cộng tuyến giữa các biến hơn, nhiều bậc tự do hơn và hiệu quả cao hơn..
- Bằng cách nghiên cứu quan sát lập đi lập lại của các số liệu chéo, số liệu mảng phù hợp hơn cho việc nghiên cứu động thái thay đổi theo thời gian của các số liệu chéo này.

- Dữ liệu bảng có thể phát hiện và đo lường tốt hơn các tác động mà người ta không thể quan sát được trong số liệu chuỗi thời gian hay số liệu chéo thuần túy
- Dữ liệu bảng làm cho chúng ta có thể nghiên cứu các mô hình hành vi phức tạp hơn.
- Bằng cách cung cấp dữ liệu đối với vài nghìn đơn vị, dữ liệu bảng có thể giảm đến mức thấp nhất hiện tượng chệch có thể xảy ra nếu chúng ta gộp phần tử theo những biến số có mức tổng hợp cao.
- Đa cộng tuyến giữa biến X_{it} và biến trễ (lag) X_{it-1} có thể được giảm bớt nhờ dữ liệu bảng. Dữ liệu bảng **lớn** thuận lợi cho việc phân tích các mô hình động (dynamic panel data)

1.5 Thuận lợi và khó khăn của mô hình phân tích số liệu mảng

Thuận lợi của số liệu mảng là cung cấp một cái nhìn đầy đủ về tất cả các tác động (cố định, ngẫu nhiên) có thể có xảy ra trên mỗi phần tử theo thời điểm và thời gian cũng như các giả thuyết thường xảy ra trong mô hình hồi quy đa biến.

Khó khăn của số liệu mảng là có khá nhiều phương pháp ước lượng có thể sử dụng được (Pooled OLS, FEM, REM, GLS, GMM,SGMM,…) để khắc phục các khuyết tật trong mô hình như phương sai thay đổi, hiện tượng đa cộng tuyến

1.6 Nguồn gốc sự thay đổi trong số liệu mảng

Time unit (t)	Cross section unit (i)					
	1	2	3	…	N	$\bar{X}_{.t}$
1	X_{11}	X_{21}	X_{31}	…	X_{N1}	$\bar{X}_{.1}$
2	X_{12}	X_{22}	X_{32}	…	X_{N2}	$\bar{X}_{.2}$
3	X_{13}	X_{23}	X_{33}	…	X_{N3}	$\bar{X}_{.3}$
…	…	…	…	…	…	…
T	X_{1T}	X_{2T}	X_{3T}	…	X_{NT}	$\bar{X}_{.T}$
$\bar{X}_{i.}$	$\bar{X}_{1.}$	$\bar{X}_{2.}$	$\bar{X}_{3.}$	…	$\bar{X}_{N.}$	$\bar{X}$

$$\overline{X}_{i.} = \frac{\sum_{t=1}^{T} X_{it}}{T}; \overline{Y}_{i.} = \frac{\sum_{t=1}^{T} Y_{it}}{T} \qquad \overline{X}_{.t} = \frac{\sum_{i=1}^{N} X_{it}}{N}; \overline{Y}_{.t} = \frac{\sum_{i=1}^{N} Y_{it}}{N}$$

$$\overline{X}_{..} = \frac{\sum_{i=1}^{N}\sum_{t=1}^{T} X_{it}}{NT}; \overline{Y}_{..} = \frac{\sum_{i=1}^{N}\sum_{t=1}^{T} Y_{it}}{NT};$$

The within-entity variation for a particular cross section unit i for entity X

$$S_{XX_i}^{W} = \sum_{t=1}^{T}\left(X_{it} - \overline{X_{i.}}\right)^2$$

For all cross section unit, the sum of squares in measuring the within-entity variation of X

$$S_{XX}^{W} = \sum_{i=1}^{N}\sum_{t=1}^{T}\left(X_{it} - \overline{X_{i.}}\right)^2$$

Covariance betweeen two variables X and Y within a particular cross section unit i

$$S_{XY_i}^{W} = \sum_{t=1}^{T}\left(X_{it} - \overline{X_{i.}}\right)\left(Y_{it} - \overline{Y_{i.}}\right)$$

Covariance betweeen two variables X and Y within group for all cross section unit

$$S_{XY}^{W} = \sum_{i=1}^{N}\sum_{t=1}^{T}\left(X_{it} - \overline{X_{i.}}\right)\left(Y_{it} - \overline{Y_{i.}}\right)$$

The sum of square measuring between-entity variation of a variable X

$$S_{XX}^{B}=\sum_{i=1}^{N}\sum_{t=1}^{T}\left(\overline{X_{i}}-\overline{X_{..}}\right)^{2}=T\sum_{i=1}^{N}\left(\overline{X_{i}}-\overline{X_{..}}\right)^{2}$$

Covariance of two variables between groups

$$S_{XY}^{B}=\sum_{i=1}^{N}\sum_{t=1}^{T}\left(\overline{X_{i}}-\overline{X_{..}}\right)\left(\overline{Y}_{i}-\overline{Y}_{..}\right)=T\sum_{i=1}^{N}\left(\overline{X_{i}}-\overline{X_{..}}\right)\left(\overline{Y}_{i}-\overline{Y}_{..}\right)$$

Total variation of X

$$S_{XX}^{T}=\sum_{i=1}^{N}\sum_{t=1}^{T}\left(X_{it}-\overline{X_{..}}\right)^{2}=S_{XX}^{W}+S_{XX}^{B}$$

Total variation between X and Y

$$S_{XY}^{T}=\sum_{i=1}^{N}\sum_{t=1}^{T}\left(X_{it}-\overline{X_{..}}\right)\left(Y_{it}-\overline{Y}_{..}\right)$$

Remark.

Panel data *(longtitudinal)* set is one that follows a given sample of individuals over time, and thus provides multiple observations on each individual in the sample (Hsiao 2003, page 2).

1.7 Panel data (balanced & unbalanced)

A panel is said to be **balanced** if we have the same time periods, t = 1,.., T, for each cross section observation. For an **unbalanced** panel, the time dimension, denoted T_i, is specific to each individual

STT	YEAR	ROAA LNTT/TTS tb
1	2009	0.020624455
1	2010	0.020498197
1	2011	0.010096679
2	2009	0.020778164
2	2010	0.016634751
2	2011	0.017290683

STT	YEAR	ROAA LNTT/TTS tb
1	2009	Null
1	2010	0.020498197
1	2011	0.010096679
2	2009	0.020778164
2	2010	Null
2	2011	0.017290683

STT	YEAR	ROAA LNTT/TTS tb
1	2009	0.020624455
1	2010	0.020498197
2	2009	0.020778164
2	2010	0.016634751
2	2011	0.017290683

1.8 Micro-panel and Macro-panel data set

Micro-panel data set is a panel for which the time dimension T is largely less important than the individual dimension N (T << N)

Disadvantge. The heterogeneity issue cannot be tackled with if the time dimension is too small

Macro-panel data set is a panel for which the time dimension T is similar to the individual dimension N (T ~ N)

1.9 Main advantage of panel data

Adv 1. The phantasm of a larger number of observation

Adv 2. New economic questions (identification)

Adv 3. Unobservable components

Adv 4. Easier estimation and inference

Adv 3. Unobservable components

$$y_{it} = \alpha + \beta' x_{it} + \rho' z_{it} + u_{it} \text{ , } i = 1, .., N \ t = 1, .., T \quad (1.1)$$

here,

x_{it} and z_{it} are k_1x1 and k_2x1 vectors of exogenous variables

α is a constant, β and ρ are k_1x1 and k_2x1 vectors of parameters

u_{it} is i.i.d. over i and t, with $V(u_{it}) = \sigma^2_u$

Let us assume that z_{it} variables unobservable and correlated with x_{it}

$$Cov(z_{it}, x_{it}) \neq 0$$

Pre Eq. (1.1) $y_{it} = \alpha + \beta' x_{it} + \mu_{it}$

It is well known that the least-squares regression coffcients of y_{it} on x_{it} are biased (endogenity bias)

Adv 3. Unobservable components (tt)

$$y_{it} = \alpha + \beta' x_{it} + \rho' z_{it} + u_{it}\ ,\ i = 1, .., N\ t = 1, ..,T \quad (1.1)$$

Method 1. Let us assume that $z_{it} = z_i$, i.e. z values stay constant through time for a given individual but vary across individuals (individual effects)

$$y_{it} = \alpha + \beta' x_{it} + \rho' z_i + u_{it} \quad (1.2)$$

$$y_{it} = \alpha + \beta' x_{it} + \mu_{it}\ ,\ \ cov\ (x_{it,}\ \mu_{it}) \neq 0 \ (1.3)$$

Then, if we take the first difference of individual observations over time:

$$y_{it} - y_{i,t-1} = \beta'(x_{it} - x_{i,t-1}) + u_{it} - u_{i,t-1} \quad (1.4)$$

Least squares regression Eq. (1.4) now provides unbiased and consistent estimates of β.

Homework. Prove Eq. (1.4) is unbiased when applying regressor by OLS

Adv 3. Unobservable components (tt)

$$y_{it} = \alpha + \beta' x_{it} + \rho' z_{it} + u_{it}\ ,\ i = 1, .., N\ t = 1, .., T \quad (1.1)$$

Method 1. Let us assume that $z_{it} = z_t$, i.e. z values are common for all individuals but vary across time (common factors)

$$y_{it} = \alpha + \beta' x_{it} + \rho' z_t + u_{it} \quad (1.5)$$

$$y_{it} = \alpha + \beta' x_{it} + \mu_{it}\ ,\ \ cov\ (x_{it}, \mu_{it}) \neq 0\ (1.6)$$

Then, if we consider deviation from the mean across individuals at a given time

$$y_{it} - \overline{y_t} = \beta'\left(x_{it} - \overline{x_t}\right) + \left(u_{it} - \overline{u_t}\right) \quad (1.7)$$

Least squares regression now provides unbiased and consistent estimates of β

Homework. Prove Eq. (1.7) is unbiased when applying regressor by OLS

1.10 Panel Data Model

$$y_{it} = \alpha_{it} + \beta'_{it} x_{it} + u_{it} \quad (1.8)$$

where

- $i = 1, .., N; t = 1, ..,T$
- α_{it} is a scalar that varies across i and t,
- $\beta_{it} = (\beta_{1it}, \beta_{2it}, ..., \beta_{Kit})'$ is a Kx1 vector of parameters that vary across i and t,
- $x_{it} = (x_{1it}, ..., x_{Kit})'$ is a Kx1 vector of exogenous variables,
- u_{it} is an error term over times (t) and individuals (i).

Remark. Model (1.8) has a large complex don't possible to research at the moment.

~~$y_{it} = \alpha_{it} + \beta'_{it} x_{it} + u_{it} \quad (1.8)$~~

In a study or finding out about panel model, Model (1.8) is usually approaching the suitable way, such as

$$y_{it} = \alpha + \beta' x_{it} + u_{it} \quad (1.8)$$

Time unit (t)	Cross section unit (i)					
	1	2	3	…	N	$\bar{X}_{.t}$
1	X_{11}	X_{21}	X_{31}	…	X_{N1}	$\bar{X}_{.1}$
2	X_{12}	X_{22}	X_{32}	…	X_{N2}	$\bar{X}_{.2}$
3	X_{13}	X_{23}	X_{33}	…	X_{N3}	$\bar{X}_{.3}$
…	…	…	…	…	…	…
T	X_{1T}	X_{2T}	X_{3T}	…	X_{NT}	$\bar{X}_{.T}$
$\bar{X}_{i.}$	$\bar{X}_{1.}$	$\bar{X}_{2.}$	$\bar{X}_{3.}$	…	$\bar{X}_{N.}$	$\bar{X}$

i	t	Y	X_1	X_2	X_3
1	1	$y_{i=1,t=1}$	$x_{1,i=1,t=1}$	x_{211}	x_{313}
1	2	y_{12}	x_{112}	x_{212}	x_{313}
1	3	y_{13}	x_{113}	x_{213}	x_{313}
2	1	y_{21}	x_{121}	x_{221}	x_{321}
2	2	y_{22}	x_{122}	x_{222}	x_{322}
2	3	y_{23}	x_{123}	x_{223}	x_{323}

$$y_{it} = \alpha + \beta' x_{it} + u_{it} \qquad (1.8)$$

Vector form Eq (1.8)

$$y_i = \begin{pmatrix} y_{i1} \\ y_{i2} \\ \dots \\ y_{iT} \end{pmatrix}_{T\times 1} ; X_i = \begin{pmatrix} x_{1,i,1} & x_{2i1} & \dots & x_{Ki1} \\ x_{1i2} & x_{2i2} & \dots & x_{Ki2} \\ \dots & \dots & \dots & \dots \\ x_{1iT} & x_{2iT} & \dots & x_{KiT} \end{pmatrix}_{T\times K} ; \beta = \begin{pmatrix} \beta_1 \\ \beta_2 \\ \dots \\ \beta_K \end{pmatrix}_{K\times 1}$$

Let us denote e a unit vector and u_i the vector of errors

$$e_i = \begin{pmatrix} 1 \\ 1 \\ \dots \\ 1 \end{pmatrix}_{T\times 1} ; u_i = \begin{pmatrix} u_{i1} \\ u_{i2} \\ \dots \\ u_{iT} \end{pmatrix}_{T\times 1}$$

1.10 Error component model

$$y_{it} = \alpha + \beta' x_{it} + u_{it} \qquad (1.8)$$

here

u_{it} is an error term over times (t) and individuals (i)

$u_{it} = \alpha_i + \lambda_t + \varepsilon_{it}$

Eq. (1.8) can rewrite as

$$y_{it} = \alpha + \beta' x_{it} + \alpha_i + \lambda_t + \varepsilon_{it} \quad (1.9)$$

Case 1. $y_{it} = \alpha + \beta' x_{it} + \alpha_i + \varepsilon_{it}$

Case 2. $y_{it} = \alpha + \beta' x_{it} + \lambda_t + \varepsilon_{it}$

Case 3. $y_{it} = \alpha + \beta' x_{it} + \alpha_i + \lambda_t + \varepsilon_{it}$

1.10.1 Pooled Ordinary Least Square (Pooled OLS)

$$y_{it} = \alpha_0 + \alpha_i + \beta' x_{it} + \varepsilon_{it.} \quad (1.10)$$

Assumption. Both slope and intercept coefficients are the same

Pre Eq. (1.10) as

$$y_{it} = \alpha + \beta' x_{it} + \varepsilon_{it.} \quad (1.11)$$

This model (1.11) is called as Pooled Ordinary Least Square

1.10.2 Fix Effect Model (FEM) & Random Effect Model (REM)

$$y_{it} = \alpha_0 + \alpha_i + \beta' x_{it} + \varepsilon_{it.} \quad (1.10)$$

In Eq. (1.10), α_i is called a **"random effect"** when it is treated as **a random variable** and a **"fix effect"** when it is treated as a parameter to be estimated for each cross section observation i.

1.10.2 Fix Effect Model (FEM) & Random Effect Model (REM)

$$y_{it} = \alpha_0 + \alpha_i + \beta' x_{it} + \varepsilon_{it.} \quad (1.10)$$

In Eq. (1.10) the term "random effect" is also synonymous with zero correlation between the observed explanatory variables and the unobserved (random) effect α_i

$$Cov(\alpha_i , x_{it}) = 0$$

In Eq. (1.10) , the term "fix effect" is allow for arbitrary correlation between the unobserverd effect α_i and the observed explanatory variables x_{it}

$$Cov(\alpha_i , x_{it}) \neq 0$$

Example 1.2 Let us consider the case of a Cobb Douglas production function in log, as defined previously, for the case T = 3 and K = 2. We have

$$y_{it} = \alpha_0 + \alpha_i + \beta_k k_{it} + \beta_n n_{it} + \varepsilon_{it.} \quad (i, t=1,..,3)$$

or in a vectorial form for a country i as:

$$y_i = e\alpha_i + X_i\beta + \varepsilon_i$$

$$\begin{pmatrix} y_{i1} \\ y_{i2} \\ y_{i3} \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \alpha_i + \begin{pmatrix} k_{i1} & n_{i1} \\ k_{i2} & n_{i2} \\ k_{i3} & n_{i3} \end{pmatrix} \begin{pmatrix} \beta_k \\ \beta_n \end{pmatrix} + \begin{pmatrix} \varepsilon_{i1} \\ \varepsilon_{i2} \\ \varepsilon_{i3} \end{pmatrix}$$

It is also possible to stackle all these vectors/matrices as follows

$$Y = \tilde{e}\tilde{\alpha} + X\beta + \varepsilon$$

$$Y_{(Tn\times 1)} = \begin{pmatrix} y_1 \\ y_2 \\ \dots \\ y_n \end{pmatrix}; \quad ;X_{(Tn\times K)} = \begin{pmatrix} X_1 \\ X_2 \\ \dots \\ X_n \end{pmatrix}; \quad ;\varepsilon_{(Tn\times 1)} = \begin{pmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \dots \\ \varepsilon_n \end{pmatrix}$$

where 0_T is the null vector (T, 1)

$$\tilde{e}_{(Tn\times n)} = I_N \otimes e = \begin{pmatrix} e & 0_T & \dots & 0_T \\ 0_T & e & \dots & 0_T \\ \dots & \dots & \dots & \dots \\ 0_T & 0_T & \dots & e \end{pmatrix}; \quad ;\tilde{\alpha}_{(n\times 1)} = \begin{pmatrix} \alpha_1 \\ \alpha_2 \\ \dots \\ \alpha_n \end{pmatrix}$$

Homework. Pre Ex. (1.2) you can arrange the stack of all these vectors/matrix with $T = 3$, $n=2$.

Remark. You should show a way detail.

1.11 Example (Bank profit and Risk)

Case 1. Static Panel Model

Chủ đề 1. *Tác động của đa dạng hóa thu nhập đến lợi nhuận và rủi ro của các Ngân hàng thương mại Việt Nam*

$$Profitability_{it} = \alpha_0 + \alpha_2 HHI\_REV_{it} + \alpha_3 L\_A_{it} + \alpha_4 SIZE_{it} + \alpha_5 ASSET\_GRO_{it} + \alpha_6 GDP\_GRO_{it} + \alpha_7 INFit + \varepsilon_{it}$$

$$Risk_{it} = \beta_0 + \beta_2 HHI\_REV_{it} + \beta_3 L\_A_{it} + \beta_4 SIZE_{it} + \beta_5 ASSET\_GRO_{it} + \beta_6 GDP\_GRO_{it} + \beta_7 INF_{it} + \varepsilon_{it}$$

Case 2. Dynamic Panel Model

Chủ đề 1. *Tác động của đa dạng hóa thu nhập đến lợi nhuận và rủi ro của các Ngân hàng thương mại Việt Nam*

$$Profitability_{it} = \alpha_0 + \alpha_1 Profitability_{it-1} + \alpha_2 HHI\_REV_{it} + \alpha_3 L\_A_{it} + \alpha_4 SIZE_{it} + \alpha_5 ASSET\_GRO_{it} + \alpha_6 GDP\_GRO_{it} + \alpha_7 INFit + \varepsilon_{it}$$

$$Risk_{it} = \beta_0 + \beta_1 Risk_{it-1} + \beta_2 HHI\_REV_{it} + \beta_3 L\_A_{it} + \beta_4 SIZE_{it} + \beta_5 ASSET\_GRO_{it} + \beta_6 GDP\_GRO_{it} + \beta_7 INF_{it} + \varepsilon_{it}$$

Chapter 2

POOLED ODINARY LEAST SQUARE (POOLED OLS)

2.1 Pooled Ordinary Least Square (Pooled OLS)

$$y_{it} = \alpha + \beta' x_{it} + \varepsilon_{it.} \quad (1.10)$$

In this model, both slope and intercept coefficients are the same

Notes. Pooled OLS does not mention the change of observation explanatory variables in both times (t=1,..,T) and cross-section unit (i =1,…,n) that can affect to the role of the observed explanatory variables.

Example 2.1 Let us consider the case of a Cobb Douglas production function in log by Pooled OLS, as defined previously, for the case T = 3 ,K = 2 and sample size (n = 2). We have

$$y_{it} = \alpha + \beta' x_{it} + \varepsilon_{it.} \quad (i, t=1,..,3)$$

$$y_i = \begin{pmatrix} y_{i1} \\ y_{i2} \\ y_{i3} \end{pmatrix}_{3\times1} ; X_i = \begin{pmatrix} x_{1i1} & x_{2i1} \\ x_{1i2} & x_{2i2} \\ x_{1i3} & x_{2i3} \end{pmatrix}_{3\times2} ; \beta = \begin{pmatrix} \beta_1 \\ \beta_2 \end{pmatrix}_{2\times1}$$

$$y_i = \alpha + X_i\beta + \varepsilon_i \quad (i = 1,2)$$

It is also possible to stackle all these vectors/matrices as follows

$$\underbrace{\begin{pmatrix} y_{11} \\ y_{12} \\ y_{13} \\ y_{21} \\ y_{22} \\ y_{23} \end{pmatrix} = \alpha + \begin{pmatrix} x_{111} & x_{211} \\ x_{112} & x_{212} \\ x_{113} & x_{213} \\ x_{121} & x_{221} \\ x_{122} & x_{222} \\ x_{123} & x_{223} \end{pmatrix} \begin{pmatrix} \beta_1 \\ \beta_2 \end{pmatrix} + \begin{pmatrix} \varepsilon_{i1} \\ \varepsilon_{i2} \\ \varepsilon_{i3} \end{pmatrix}}_{\text{Pooled OLS}}$$

Giả thiết 1. Giá trị trung bình của sai số ngẫu nhiên (SSNN) bằng 0, nghĩa là: $\mathbf{E(\varepsilon/X= x_i) = 0}$

Giả thiết 2. Phương sai của các SSNN bằng nhau, nghĩa là:

$\mathbf{Var(\varepsilon/X= x_i) = \sigma^2}$

Giả thiết 3. Các SSNN không tương quan với nhau, $\mathbf{Cov(\varepsilon_i , \varepsilon_j) = 0}$

Giả thiết 4. Các SSNN và biến giải thích (X_i) không tương quan với nhau, nghĩa là: $\mathbf{Cov(\varepsilon_i , X_i) = 0}$

Giả thiết 5. Các biến giải thích (X_1, …, X_k) độc lập tuyến tính

Homework. Using file "Data_Ch1.xlsx" to run the following model

$$Profitability_{it} = \alpha_0 + \alpha_2 HHI\_REV_{it} + \alpha_3 L\_A_{it} + \alpha_4 SIZE_{it} + \alpha_5 ASSET\_GRO_{it} + \alpha_6 GDP\_GRO_{it} + \alpha_7 INFit + \varepsilon_{it}$$

by applying Pooled OLS and answer these questions.

(1) Test significance of parameters ahead of variables

(2) Test all hypotheses to ensure the result running by Pooled OLS is unbiased.

(3) Explain the significance of the explanatory variables that our p_value are accept

Chapter 3

Fix Effect Model (FEM)

Objectives

(1) Introduce about Fix Effect Model

(2) Estimates the slope paramaters in FEM by Within Estimator, Between Estimator

(3) Estimates FEM by Least Square Dummy Variables (LSDV) method

3.1 Introduce about FEM

Notations

Let us denote

$$y_i = \begin{pmatrix} y_{i1} \\ y_{i2} \\ \dots \\ y_{iT} \end{pmatrix}_{T\times 1} ; X_i = \begin{pmatrix} x_{1,i,1} & x_{2i1} & \dots & x_{Ki1} \\ x_{1i2} & x_{2i2} & \dots & x_{Ki2} \\ \dots & \dots & \dots & \dots \\ x_{1iT} & x_{2iT} & \dots & x_{KiT} \end{pmatrix}_{T\times K} ; \beta = \begin{pmatrix} \beta_1 \\ \beta_2 \\ \dots \\ \beta_K \end{pmatrix}_{K\times 1}$$

Let us denote e a unit vector and ε_i the vector of errors

$$e = \begin{pmatrix} 1 \\ 1 \\ \dots \\ 1 \end{pmatrix}_{T\times 1} ; \varepsilon_i = \begin{pmatrix} \varepsilon_{i1} \\ \varepsilon_{i2} \\ \dots \\ \varepsilon_{iT} \end{pmatrix}_{T\times 1}$$

We consider the fix effect model:

$$y_i = \alpha_0 + e\alpha_i + X_i\beta + \varepsilon_{i.}\ (i = 1, .., n) \quad (3.1)$$

where

α_i is assumed to be a constant term or have correlation with the explanatory variables

Assumption 3.1 The error term ε_{it} are i.i.d ($\forall$it) with:

- $E(\varepsilon_{it}) = 0$
- $E(\varepsilon_{it}\varepsilon_{is}) = \sigma^2_\varepsilon$ when t =s and = 0 if $t \neq s$ or $E(\varepsilon_i\varepsilon'_i) = \sigma^2_\varepsilon I_T$ here I_T denotes the identity matrix (T,T)
- $E(\varepsilon_{it}\varepsilon_{js}) = 0$, $\forall\ i \neq j$, $\forall$(ts), or $E(\varepsilon_i\varepsilon'_j) = 0_T$ here 0_T denotes the identity matrix (T,T)

Theorem 3.1 Under assumption (3.1), OLS estimator of parameters (β) is the best linear unbiased estimator (BLUE)

3.2 Estimates the slope paramaters

Case 1. Single regression

Method 1. Within Estimator

$y_i = e\alpha_i + X_i\beta + \varepsilon_{i.}$ $(i = 1, .., n)$ (3.1)

$y_{it} = \alpha_i + x_{it}\beta + \varepsilon_{it}$ $(\forall it)$ (3.1)

Taking mean of this equation (3.1) over time for each cross section unit i, we have

$$\overline{y}_i = \alpha_i + \overline{x}_i\beta + \overline{\varepsilon}_i \quad (3.2)$$

Again by taking average Eq. (3.2) across individuals, we have

$$\overline{y}_{..} = \alpha_i + \overline{x}_{..}\beta + \overline{\varepsilon}_{..} \quad (3.3)$$

Subtracting Eq. (3.2) from Eq. (3.1) for each t to get

$$\left(y_{it} - \overline{y}_i\right) = \beta\left(x_{it} - \overline{x}_i\right) + \left(\varepsilon_{it} - \overline{\varepsilon}_i\right) \quad (3.4)$$

Remark. (3.4) can be estimated by applying OLS, also calling the name "**Within Estimator**"

$$\hat{\beta}^{W} = \frac{S_{xy}^{W}}{S_{xx}^{W}}; (3.5) \qquad RSS^{W} = S_{yy}^{W} - \hat{\beta}^{'} S_{xy}^{W}$$

Method 2. Between estimator (BE)

Subtracting (3.2) from (3.3) for each t to get

$$\left(\overline{y_{i}} - \overline{y_{..}}\right) = \beta\left(\overline{x_{i}} - \overline{x_{..}}\right) + \left(\overline{\varepsilon_{i}} - \overline{\varepsilon_{..}}\right) \qquad (3.6)$$

Between estimation (3.6) by OLS

$$\hat{\beta}^{B} = \frac{S_{xy}^{B}}{S_{xx}^{B}}; (3.7)$$

Example 3.1 Using file "Data_Ch1.xlsx" to run the following model

$$Profitability_{it} = \alpha_0 + \alpha_2 HHI\_REV_{it} + \varepsilon_{it}$$

Remark. OLS estimates by Pooled OLS can be looked as the weight sum of within estimates and between estimates

$$\hat{\beta}^{Pooled\ OLS} = \frac{S_{xy}^{Pool=T}}{S_{xx}^{Pool=T}} = \frac{S_{xy}^{W} + S_{xy}^{B}}{S_{xx}^{T}} = \hat{\beta}^{W} F_{xx}^{W} + \hat{\beta}^{B} F_{xx}^{B}$$

Case 2. Multiple regression

Method 1. Within estimation

$$y_{it} = \alpha_i + x_{it}\beta + \varepsilon_{it} \qquad (3.8)$$

here

- $\beta' = (\beta_1, \beta_2, \ldots, \beta_k)$;
- $x'_{it} = (x_{1it}, x_{2it}, \ldots, x_{kit})$;
- α_i is a scalar intercepts representing the unobserved effects which are same over time;

- The error term, ε_{it}, represents the effects of omitted variables that will change across the individual units and time periods.

Assumption. ε_{it} is not uncorrelate with x_{it} and $\varepsilon_{it} \sim N(0, \sigma^2_\varepsilon)$

In vector form, (3.8) can expressed for unit *i* as

$$\begin{pmatrix} y_{i1} \\ y_{i2} \\ \dots \\ y_{iT} \end{pmatrix} = \begin{pmatrix} \alpha_i \\ \alpha_i \\ \dots \\ \alpha_i \end{pmatrix} + \begin{pmatrix} x_{1i1} & x_{2i1} & \dots & x_{ki1} \\ x_{1i2} & x_{2i2} & \dots & x_{ki2} \\ \dots & \dots & \dots & \dots \\ x_{1iT} & x_{2iT} & \dots & x_{kiT} \end{pmatrix} \begin{pmatrix} \beta_1 \\ \beta_2 \\ \dots \\ \beta_k \end{pmatrix} + \begin{pmatrix} \varepsilon_{i1} \\ \varepsilon_{i2} \\ \dots \\ \varepsilon_{iT} \end{pmatrix}$$

Or,

$$y_i = e\alpha_i + X_i\beta + \varepsilon_i \qquad (3.9)$$

e is a vector of oder T, e' = (1, 1, …,1)

Set $Q = I_T - \frac{1}{T}ee'$

Pre multiplying **Eq. (3.9)** by Q, we have

$$Qy_i = Qe\alpha_i + QX_i\beta + Q\varepsilon_i$$

Now,

$$Qy_i = \left(I_T - \frac{1}{T}ee'\right)y_i = \begin{pmatrix} 1-\frac{1}{T} & -\frac{1}{T} & \dots & -\frac{1}{T} \\ -\frac{1}{T} & 1-\frac{1}{T} & \dots & -\frac{1}{T} \\ \dots & \dots & \dots & \dots \\ -\frac{1}{T} & -\frac{1}{T} & \dots & 1-\frac{1}{T} \end{pmatrix} \begin{pmatrix} y_{i1} \\ y_{i2} \\ \dots \\ y_{iT} \end{pmatrix} = \begin{pmatrix} y_{i1} - \overline{y_{i.}} \\ y_{i2} - \overline{y_{i.}} \\ \dots \\ y_{iT} - \overline{y_{i.}} \end{pmatrix}$$

Eq. (3.9) can show that Qe =0 → $Qy_i = QX_i\beta + Q\varepsilon_i$ (3.10)

We can apply OLS to find β parameter of Eq. (3.10)

$$\widehat{\beta}_W = \sum_{i=1}^{N}\left((QX_i)'(QX_i)\right)^{-1}(QX_i)'(Qy_i) = \sum_{i=1}^{N}\left(X_i'QX_i\right)^{-1}\left(X_i'Qy_i\right) \quad (3.11)$$

For all cross section units N and over time T, Eq. (3.10) can be expressed in the following matrix form:

$$QY = QD\alpha + QX\beta + Q\varepsilon = QX\beta + Q\varepsilon \quad (3.12)$$

Here,

$$Y = \begin{pmatrix} y_1 \\ y_2 \\ \ldots \\ y_N \end{pmatrix}, D = \begin{pmatrix} e & 0 & \ldots & 0 \\ 0 & e & \ldots & 0 \\ \ldots & \ldots & \ldots & \ldots \\ 0 & 0 & \ldots & e \end{pmatrix}, X = \begin{pmatrix} X_1 \\ X_2 \\ \ldots \\ X_N \end{pmatrix}, \varepsilon = \begin{pmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \ldots \\ \varepsilon_N \end{pmatrix}$$

The OLS obtained from Eq. (3.12) is

$$\widehat{\beta_W} = \left[(QX)'(QX)^{-1}\right](QX)'(QY) = (X'QX)^{-1}(X'QY)$$

$$= \sum_{i=1}^{N}\left(X_i'QX_i\right)^{-1}\left(X_i'Qy_i\right) \qquad (3.13)$$

Substituting (3.13) into (3.12)

$$\widehat{\beta_W} = (X'QX)^{-1}(X'QY) = (X'QX)^{-1}\left(X'Q(X\beta+\varepsilon)\right)$$

$$= \beta + (X'QX)^{-1}X'Q\varepsilon \qquad (3.14)$$

Therefore, $E\left(\widehat{\beta_W}\right) = \beta; \quad Var\left(\widehat{\beta_W}\right) = \sigma_\varepsilon^2\left[\sum_{i=1}^{N} X_i'QX_i\right]^{-1} = \sigma_\varepsilon^2(X'QX)^{-1}$

Pre Example 3.1 With model

ROAA = f(HHI, L_A, SIZE, ASSET_GRO, GDP, INF) + ε

Requirement:

- The within estimates by OLS
- The between estimates by OLS

Remark. The within estiamates can also obtained by panel regression by using xtreg command in Stata with option fixed efects denoting by fe

3.3 Least Squares Dummy Variable (LSDV) Regression

$$y_{it} = \alpha_i + x_{it}\beta + \varepsilon_{it}$$

$$\begin{pmatrix} y_{i1} \\ y_{i2} \\ \dots \\ y_{iT} \end{pmatrix} = \begin{pmatrix} \alpha_i \\ \alpha_i \\ \dots \\ \alpha_i \end{pmatrix} + \begin{pmatrix} x_{1i1} & x_{2i1} & \dots & x_{ki1} \\ x_{1i2} & x_{2i2} & \dots & x_{ki2} \\ \dots & \dots & \dots & \dots \\ x_{1iT} & x_{2iT} & \dots & x_{kiT} \end{pmatrix} \begin{pmatrix} \beta_1 \\ \beta_2 \\ \dots \\ \beta_k \end{pmatrix} + \begin{pmatrix} \varepsilon_{i1} \\ \varepsilon_{i2} \\ \dots \\ \varepsilon_{iT} \end{pmatrix}$$

Or,

$$y_i = e\alpha_i + X_i\beta + \varepsilon_i \qquad (3.9)$$

In vector form, for all cross section units , Eq. (3.9) can be expressed as

$$\begin{pmatrix} y_1 \\ y_2 \\ \dots \\ y_N \end{pmatrix} = \begin{pmatrix} e \\ 0 \\ \dots \\ 0 \end{pmatrix} \alpha_1 + \begin{pmatrix} 0 \\ e \\ \dots \\ 0 \end{pmatrix} \alpha_2 + \dots + \begin{pmatrix} 0 \\ 0 \\ \dots \\ e \end{pmatrix} \alpha_N + \begin{pmatrix} X_1 \\ X_2 \\ \dots \\ X_N \end{pmatrix} \begin{pmatrix} \beta_1 \\ \beta_2 \\ \dots \\ \beta_k \end{pmatrix} + \begin{pmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \dots \\ \varepsilon_N \end{pmatrix}$$

Here,

$$y_i = \begin{pmatrix} y_{i1} \\ y_{i2} \\ \dots \\ y_{iT} \end{pmatrix}_{T\times 1} ; e = \begin{pmatrix} 1 \\ 1 \\ \dots \\ 1 \end{pmatrix}_{T\times 1} ; X_i = \begin{pmatrix} x_{1i1} & x_{2i1} & \dots & x_{ki1} \\ x_{1i2} & x_{2i2} & \dots & x_{ki2} \\ \dots & \dots & \dots & \dots \\ x_{1iT} & x_{2iT} & \dots & x_{kiT} \end{pmatrix}_{T\times K} ; \varepsilon = \begin{pmatrix} \varepsilon_{i1} \\ \varepsilon_{i2} \\ \dots \\ \varepsilon_{iT} \end{pmatrix}_{T\times 1}$$

Eq. reduces to $Y = \alpha' D + X\beta + \varepsilon$

Here, D is the NT x N matrix for dummy regressor and can be expressed as $D = I_N \otimes e_T$

Kronecker product $A\otimes B$ of two matrices $A = (a_{ij})_{nm}$ and $B = (b_{kl})_{n1m1}$ is defined by

$$A\otimes B = \begin{pmatrix} a_{11}B & a_{12}B & \dots & a_{1m}B \\ a_{21}B & a_{22}B & \dots & a_{2m}B \\ \dots & \dots & \dots & \dots \\ a_{n1}B & a_{n2}B & \dots & a_{nm}B \end{pmatrix}_{nn_1\times mm_1}$$

$$(A\otimes B)' = A'\otimes B'$$

$$(A\otimes B)^{-1} = A^{-1}\otimes B^{-1}$$

$$(A\otimes B)(C\otimes D) = AC\otimes BD$$

Assumption 3.1 The error term ε_{it} are i.i.d ($\forall$it) with:

- E (ε_{it}) = 0
- E ($\varepsilon_{it}\varepsilon_{is}$) = σ^2_ε when t =s and = 0 if t $\neq$ s or E($\varepsilon_i\varepsilon'_i$) = $\sigma^2_\varepsilon I_T$ here I_T denotes the identity matrix (T,T)
- E ($\varepsilon_{it}\varepsilon_{js}$) = 0 , $\forall$ i $\neq$ j, $\forall$(ts), or E($\varepsilon_i\varepsilon'_j$)= 0_T here 0_T denotes the identity matrix (T,T)

Theorem 3.1 Under assumption (3.1), OLS estimator of parameters (β) is the best linear unbiased estimator (BLUE)

The OLS estimator of α_i and β are obtained by minmising

$$S_u = \sum_{i=1}^{N} \varepsilon_i' \varepsilon_i = \sum_{i=1}^{N} (y_i - e\alpha_i - X_i\beta)' (y_i - e\alpha_i - X_i\beta) \qquad (3.15)$$

Taking partial derivates Eq. (3.15) with respect to α_i and β setting them to zero, we have

$$\varepsilon_i = \overline{y_i} - \overline{x_i}' \beta$$

$$\overline{y_i} = \frac{1}{T} \sum_{t=1}^{T} y_{it}; \qquad \overline{x_i} = \frac{1}{T} \sum_{t=1}^{T} x_{it}$$

$$\widehat{\beta_{LSDV}} = \left[\sum_{i=1}^{N} \sum_{t=1}^{T} (x_{it} - \overline{x_i})' (x_{it} - \overline{x_i}) \right]^{-1} \left[\sum_{i=1}^{N} \sum_{t=1}^{T} (x_{it} - \overline{x_i})' (y_{it} - \overline{y_i}) \right] = \frac{S_{xy}^{W}}{S_{xx}^{W}}$$

Pre Example (3.1) With model

ROAA = f(HHI, L_A, SIZE, ASSET_GRO, GDP, INF) + ε

Remark. There are too many parameters in the fixed effects model and the loss of degrees of freedom can be avoided if the α_i can be assumed random

Homework. With three research include:

1. Grunfeld Investment Equation (p.21)

2. Gasoline Demand (p.23)

3. Public Capital Productivity (p.25)

a) Estimate parameters ahead of the explanatory variables in those study by Within estimator, Between estimator, LSDV and FEM by command xtreg.

b) Comparing results receiving from those models

c) Explanatory about α_i parameters in method LSDV

Chapter 4

Random effect model (REM)

Objectives

(1) Introduce about Random Effect Model

(2) Estimates the slope paramaters in FEM by Within Estimator, Between Estimator

(3) Estimates FEM by Least Square Dummy Variables (LSDV) method

Notes. There are too many parameters in the fixed effects model and the loss of degrees of freedom can be avoided if the α^*_i can be assumed random

4.1 Introduce Random effect model

$$y_{it} = x_{it}\beta + \alpha^*_i + u_{it} \qquad i = \overline{1,N};\ t = \overline{1,T} \quad (4.1)$$

Here,

α^*_i is assumed to be random

If the individual effects α^*_i are supposed to have **non zero mean**, with

$E(\alpha^*_i) = \alpha_0$

Then we can define cross section units effects $\boldsymbol{\alpha^*_i = \alpha_0 + \alpha_i}$

Pre Eq. (4.1) $y_{it} = x_{it}\beta + \alpha_0 + \underbrace{\alpha_i + u_{it}}_{\varepsilon_{it}} = \alpha_0 + x_{it}\beta + \varepsilon_{it}$

4.1.1 The assumptions on the components of errors

About α_i,

$$E(\alpha_i) = 0, \quad ,V(\alpha_i) = E(\alpha_i^2) = \sigma_\mu^2, \quad ,E(\alpha_i x_{it}) = 0, \quad ,E(\alpha_i \alpha_j) = 0$$

About u_{it},

$$E(u_{it}) = 0, \quad ,V(u_{it}) = E(u_{it}^2) = \sigma_u^2, \quad ,E(u_{it} u_{js}) = 0 \text{ for } i \neq j \text{ and } t \neq s$$

The components of the error are not correlated

$$E(\alpha_i u_{it}) = 0$$

Remark. The α_i are independent of the error term u_{it} and the regressors x_{it}, for all i and t

4.1.2 Mean and variance of errors

The mean and variance of the component errors are

$$E(\varepsilon_{it}) = 0, \quad ,V(\varepsilon_{it}) = V(y_{it}) = \sigma_\alpha^2 + \sigma_u^2$$

The covariance of the composite error,

$Cov(\varepsilon_{it,} \varepsilon_{js}) = E(\varepsilon_{it}\varepsilon_{js}) = E(\alpha_i + u_{it})(\alpha_j + u_{js})$

$= E(\alpha_i\alpha_j + u_{it}\alpha_j + \alpha_i u_{js} + u_{it}u_{js})$

Or

Case 1. $Cov(\varepsilon_{it,} \varepsilon_{js}) = \sigma^2_\alpha + \sigma^2_u \quad \forall i = j, t = s$

Case 2. $Cov(\varepsilon_{it,} \varepsilon_{js}) = \sigma^2_\alpha \quad \forall i = j, t \neq s$

Case 3. $Cov(\varepsilon_{it,} \varepsilon_{js}) = 0 \quad \forall i \neq j, t \neq s$

For cross section unit i, Eq. (4.1) can be written as

$$\underbrace{\begin{pmatrix} y_{i1} \\ y_{i2} \\ \ldots \\ y_{iT} \end{pmatrix}}_{\substack{y_i \\ (T,1)}} = \underbrace{\begin{pmatrix} 1 & x_{1i1} & x_{2i1} & \ldots & x_{ki1} \\ 1 & x_{1i2} & x_{2i2} & \ldots & x_{ki2} \\ 1 & \ldots & \ldots & \ldots & \ldots \\ 1 & x_{1iT} & x_{2iT} & \ldots & x_{kiT} \end{pmatrix}}_{\substack{\widetilde{X}_i \\ (T,K+1)}} \underbrace{\begin{pmatrix} \alpha_0 \\ \beta_1 \\ \beta_2 \\ \ldots \\ \beta_k \end{pmatrix}}_{\substack{\gamma \\ (K+1,1)}} + \underbrace{\begin{pmatrix} \varepsilon_{i1} \\ \varepsilon_{i2} \\ \ldots \\ \varepsilon_{iT} \end{pmatrix}}_{\substack{\varepsilon_i \\ (T,1)}}$$

$$\Leftrightarrow y_i = \widetilde{X}_i \gamma + \varepsilon_i$$

The variance- covariance matrix of ε_i (for individual i) is

$$E\left(\varepsilon_i \varepsilon_i'\right) = E\left(\begin{pmatrix} \varepsilon_{i1} \\ \varepsilon_{i2} \\ \ldots \\ \varepsilon_{iT} \end{pmatrix} \begin{pmatrix} \varepsilon_{i1} & \varepsilon_{i2} & \ldots & \varepsilon_{iT} \end{pmatrix}\right)$$

$$= E\begin{pmatrix} \varepsilon_{i1}^2 & \varepsilon_{i1}\varepsilon_{i2} & \dots & \varepsilon_{i1}\varepsilon_{iT} \\ \varepsilon_{i2}\varepsilon_{i1} & \varepsilon_{i2}^2 & \dots & \varepsilon_{i2}\varepsilon_{iT} \\ \dots & \dots & \dots & \dots \\ \varepsilon_{iT}\varepsilon_{i1} & \varepsilon_{iT}\varepsilon_{i2} & \dots & \varepsilon_{iT}^2 \end{pmatrix}$$

$$= \begin{pmatrix} \sigma_\alpha^2 + \sigma_u^2 & \sigma_\alpha^2 & \dots & \sigma_\alpha^2 \\ \sigma_\alpha^2 & \sigma_\alpha^2 + \sigma_u^2 & \dots & \sigma_\alpha^2 \\ \dots & \dots & \dots & \\ \sigma_\alpha^2 & \sigma_\alpha^2 & \dots & \sigma_\alpha^2 + \sigma_u^2 \end{pmatrix}$$

$$= U = \sigma_u^2 I_T + \sigma_\alpha^2 ee' = \sigma_u^2\left(Q + \frac{1}{T}ee'\right) + T\sigma_\alpha^2 \frac{1}{T}ee'$$

$$= \sigma_u^2 Q + \left(\sigma_u^2 + T\sigma_\alpha^2\right)P \qquad (4.2)$$

here $P = \frac{1}{T} ee' = e\left(e'e\right)^{-1} e' = I_T - Q$

$$(4.2) \Rightarrow U^{-1} = \frac{1}{\sigma_u^2}\left(Q + \theta P\right) \qquad (4.3)$$

where $\theta = \frac{\sigma_u^2}{\left(\sigma_u^2 + T\sigma_\alpha^2\right)} \qquad (4.4)$

Therefore, $U^{-1/2} = \frac{1}{\sigma_u} Q + \frac{1}{\sqrt{\left(\sigma_u^2 + T\sigma_\alpha^2\right)}} P \quad (4.5)$

Or, $U^{-1/2} = \frac{1}{\sigma_u}\left(Q + P\sqrt{\frac{1}{\left(\sigma_u^2 + T\sigma_\alpha^2\right)}} \right) \quad (4.6)$

And $\left|U\right| = \sigma_u^{2(T-1)}\left(\sigma_u^2 + T\sigma_\alpha^2\right) \quad (4.7)$

By taking all cross section units in the sample, the variance - covariance matrix of the error term (ε) will be of order *NT x NT*

$$E\left(\varepsilon\varepsilon'\right) = E\begin{pmatrix} U & 0 & \dots & 0 \\ 0 & U & \dots & 0 \\ \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & U \end{pmatrix}$$

$$= U \otimes I_N = \sigma_u^2\left(I_T \otimes I_N\right) + \sigma_\alpha^2\left(J_T \otimes I_N\right) = \Omega \quad (4.8)$$

where $J_T = ee'$

4.2 GLS estimation

Idea. The generalised least squares (GLS) is used in estimating a random effects model when U is known.

Suppose that the variance – covariance matrix (U) is **known**

Pre multiply Eq. (4.1) by $U^{-1/2}$ to get

$$U^{-1/2}y_i = U^{-1/2}\widetilde{X}_i\gamma + U^{-1/2}\varepsilon_i$$

$$\Leftrightarrow y_i^* = X_i^*\gamma + \varepsilon_i^* \quad (4.9)$$

Where,

$$E\left(\varepsilon_i^*\varepsilon_i^{'*}\right) = E\left[\left(U^{-1/2}\varepsilon_i\right)\left(U^{-1/2}\varepsilon_i\right)'\right] = U^{-1/2}E\left(\varepsilon_i\varepsilon_i'\right)U^{-1/2} = I_T \quad (4.10)$$

Homework. Proving why reason Eq. (4.10) equal with I_T

With Eq. (4.9) we can apply OLS to estimate parameter (γ)

The GLS estimators of γ are

$$\hat{\gamma}_{GLS} = \left(\sum_{i=1}^{N} X_i^{'*} X_i^{*} \right)^{-1} \sum_{i=1}^{N} X_i^{'*} y_i^{*} \quad (4.11)$$

or

$$\hat{\gamma}_{GLS} = \left(\sum_{i=1}^{N} \widetilde{X}_i^{'} U^{-1} \widetilde{X}_i \right)^{-1} \sum_{i=1}^{N} \widetilde{X}_i^{'} U^{-1} y_i \quad (4.12)$$

or

$$\hat{\gamma}_{GLS} = \left(\widetilde{X}^{'} \Omega \widetilde{X} \right)^{-1} \left(\widetilde{X}^{'} \Omega y \right) \quad (4.13)$$

Eq. (4.12) can be written in expanded form as

$$\text{Eq. } (4.12) = \left(S_{XX}^{W} + \theta S_{XX}^{B} \right)^{-1} \left(S_{Xy}^{W} + \theta S_{Xy}^{B} \right) \quad (4.13)$$

Homework. Expanding detail (4.12) to finding why (4.12) can similar (4.13)

Remark.

- If $\theta = 1$, then GLS estimator is equivalent to OLS pooled estimator.
- If $\theta = 0$, then GLS estimator will be equal to LSDV
- The parameter θ measures **the weight** given to between-group variation.
- If U is unknown, we can use a two-step GLS estimation known with name is called FGLS (Feasible Generalized Least Squares)

4.3 FGLS estimator

Note. When U is unknown as means as σ^2_α & σ^2_u are unidentifed. We can use two-step GLS estimation known as FGLS

Step 1. We estimate the "within" estimation and "between" estimation model to find out

$$\hat{\sigma}_u^2 = \frac{\sum_{i=1}^{N}\sum_{i=1}^{T}\left[\left(y_{it}-\bar{y}_i\right)-\hat{\gamma}_W'\left(x_{it}-\bar{x}_i\right)\right]^2}{N\left(T-1\right)-K} \quad (4.14)$$

and

$$\hat{\sigma}_\alpha^2 = \frac{\sum_{i=1}^{N}\left(\bar{y}_i-\hat{\gamma}'\bar{x}_i\right)^2}{N-K} - \frac{1}{T}\hat{\sigma}_u^2 \quad (4.15)$$

The σ^2_α & σ^2_u are obtained from the "between " effect estimation, the "within" effect estimation, etc.

Then, we have to caculate

$$\hat{U}^{-1/2} = \frac{1}{\hat{\sigma}_u}Q + P\sqrt{\frac{\hat{\sigma}_u^2}{\left(\hat{\sigma}_u^2 + T\hat{\sigma}_\alpha^2\right)}} \quad (4.16)$$

Step 2. We have to estimate the following model:

$$\hat{U}^{-1/2} y_i = \hat{U}^{-1/2} \tilde{X}_i \gamma + \hat{U}^{-1/2} \varepsilon_i \quad (4.17)$$

4.4 Testing of Hypotheses

Introduction. *In a panel regression model, either fixed or random effect is an issue of unobserved variables measuring heterogeneity across the entities which renders the bias in pooled regression estimation.*

4.4.1 Measuring of Goodness Fit

Panel data can be utilities to calculate within-entity variation (R^2_W) , between-entity variation (R^2_B) and overall variation (R^2).

Option 1. Testing for Pooled Regression

$$y_i = \alpha + X_i\beta + \varepsilon_{i.} \ (i = 1, .., n)$$

Pre Example (3.1) with model

ROAA = f(HHI, L_A, SIZE, ASSET_GRO, GDP, INF) + ε

Option 2. Testing for Fix Effects

$$y_i = e\alpha_i + X_i\beta + \varepsilon_{i.}\ (i = 1, .., n)$$

Method 1. Fix effects model is only valid when we could test the joint significance of the dummies by:

H_0: $\alpha_1 = \alpha_2 = \ldots = \alpha_N = 0$

H_1: $\alpha_i \neq 0$

The F test is calculated by the following formular:

$$F = \frac{\left(\hat{\varepsilon}'\hat{\varepsilon} - \sum_{i=1}^{N}\hat{\varepsilon}_i'\hat{\varepsilon}_i\right)/(N-1)}{\sum_{i=1}^{N}\hat{\varepsilon}_i'\hat{\varepsilon}_i\ /\left(NT-K-1\right)} \sim F_{(N-1),(NT-K-1)}$$

Pre Example (3.1) With model

ROAA = f(HHI, L_A, SIZE, ASSET_GRO, GDP, INF) + ε

Method 2. F test can alse check by xtreg with option fe in Stata

Option 3. Testing of Random Effects

$$y_{it} = x_{it}\beta + \alpha_0 + \underbrace{\alpha_i + u_{it}}_{\varepsilon_{it}} = \alpha_0 + x_{it}\beta + \varepsilon_{it}$$

H_0: $\sigma^2_\alpha = 0$

H_1: $\sigma^2_\alpha > 0$

To test this hypothesis, we can use Lagrange Multiplier (LM) test developed by Bresuch and Pagan (1980)

$$LM = \frac{NT}{2(T-1)}\left[1 - \frac{\hat{\varepsilon}'(I_N \otimes J_T)\hat{\varepsilon}}{\hat{\varepsilon}'\hat{\varepsilon}}\right]^2$$

Pre Example (3.1) With model

ROAA = f(HHI, L_A, SIZE, ASSET_GRO, GDP, INF) + ε

4.4.2 Fix or Random effect: Hausman Test

$y_i = e\alpha_i + X_i\beta + \varepsilon_{i.}$ (i = 1, .., n) (FE)

$$y_{it} = x_{it}\beta + \alpha_0 + \underbrace{\alpha_i + u_{it}}_{\varepsilon_{it}} = \alpha_0 + x_{it}\beta + \varepsilon_{it}$$

H_0: $E(\varepsilon_{it}|X_{it}) = 0$

H_1: $E(\varepsilon_{it}|X_{it}) \neq 0$

$$\hat{\gamma}_{GLS} = \left(\tilde{X}'\Omega\tilde{X}\right)^{-1}\left(\tilde{X}'\Omega y\right) = \gamma + \left(\tilde{X}'\Omega\tilde{X}\right)^{-1}\left(\tilde{X}'\varepsilon\right) \quad (4.13)$$

$$\Rightarrow \hat{\beta}_{GLS} = \left(\tilde{X}'\Omega\tilde{X}\right)^{-1}\left(\tilde{X}'\Omega y\right) = \beta + \left(\tilde{X}'\Omega\tilde{X}\right)^{-1}\left(\tilde{X}'\varepsilon\right) \quad (4.18)$$

$$\hat{\beta} = \left(\tilde{X}'Q\tilde{X}\right)^{-1}\left(\tilde{X}'Qy\right) = \beta + \left(\tilde{X}'Q\tilde{X}\right)^{-1}\left(\tilde{X}'\varepsilon\right) \quad (3.13)$$

Using this fact, we have

$$\hat{\beta}_{GLS} - \beta = \left(\tilde{X}'\Omega\tilde{X}\right)^{-1}\left(\tilde{X}'\varepsilon\right) \quad (4.18)$$

$$\hat{\beta}_{W} - \beta = \left(\tilde{X}'Q\tilde{X}\right)^{-1}\left(\tilde{X}'\varepsilon\right) \quad (3.13)$$

Therefore,

With $q = \hat{\beta}_{GLS} - \hat{\beta}_{W}$

Then $E(q) = 0; \quad Var(q) = Var\left(\hat{\beta}_{GLS}\right) - Var\left(\hat{\beta}_{W}\right)$

$$Cov\left(\hat{\beta}_{GLS}, q\right) = 0 \quad (p.66)$$

$$Var(q) = Var\left(\hat{\beta}_{GLS}\right) - Var\left(\hat{\beta}_{W}\right) = Var\left(\hat{\beta}_{W}\right) - Cov\left(\hat{\beta}_{W}, \hat{\beta}_{GLS}\right) \quad (p.67)$$

The test statistic is

Hausman test (H) = $q'(Var(q))^{-1}q$

Chapter 5

Dynamic Panel Model

Objectives

(1) Introduce about Dynamic Panel Model

(2) Panel Unit Root Test

(3)

5.1 Introduction

Linear dynamic panel data models include lag dependent variables as covariates along with the unobserved effects, fixed or random, and exogenous regressor

$$y_{it} = \gamma_0 + \sum_{j=1}^{p} \gamma_j y_{t-j} + x_{it}\beta + \alpha_i + u_{it} = \sum_{j=1}^{p} \gamma_j y_{t-j} + x_{it}\beta + \alpha_i^* + u_{it} \quad (5.1)$$

Notes: *The presence of lagged dependent variable as a regressor incorporates the entire history of it, and any impact of* x_{it} *on* y_{it} *is conditioned on this history.*

We consider a dynamic panel model, in the sense that it contains (at least) one lagged variables. For simplicity, let us consider

$$y_{it} = \gamma_1 y_{it-1} + \beta'_{it} x_{it} + \alpha_i^* + u_{it} \quad (5.2)$$

$$y_{it} = \gamma_1 y_{it-1} + \beta'_{it} x_{it} + + \alpha_i^* + u_{it} \qquad (5.2)$$

Eq. (5.2) requires that $|\gamma| < 1$

$$y_{it} = \gamma_1 y_{it-1} + \alpha_i^* + u_{it} = \gamma_0 + \gamma_1 y_{it-1} + \alpha_i + u_{it} \qquad (5.3)$$

Assumptions on random disturbance are the following:

About α_i,

$$E(\alpha_i) = 0, \quad , V(\alpha_i) = E(\alpha_i^2) = \sigma_\mu^2, \quad , E(\alpha_i x_{it}) = 0, \quad , E(\alpha_i \alpha_j) = 0$$

About u_{it},

$$E(u_{it}) = 0, \quad , V(u_{it}) = E(u_{it}^2) = \sigma_u^2, \quad , E(u_{it} u_{js}) = 0 \text{ for } i \neq j \text{ and } t \neq s$$

$$E(u_{it} / y_{it-1}) = 0$$

$$E(\alpha_i / y_{it-1}) \neq 0$$

By setting t = 1, 2,… and so on, the autoregressive process can be expressed in the following way:

$$y_{i(t=1)} = \gamma_0 + \gamma y_{i0} + \alpha_i + u_{i0}$$

$$y_{i2} = \gamma_0 + \gamma y_{i1} + \alpha_i + u_{i2} = \gamma_0 + \alpha_i + \gamma_1\left(\gamma_0 + \gamma_1 y_{i0} + \alpha_i + u_{i0}\right) + u_{i2}$$

$$= \gamma_0 + \gamma_0\gamma_1 + \alpha_i + \alpha_i\gamma_1 + \gamma_1^2 y_{i0} + \gamma_1 u_{i1} + u_{i2}$$

....

$$y_{it} = \gamma_0\left(1 + \gamma_1 + \ldots + \gamma_1^{t-1}\right) + \alpha_i\left(1 + \gamma_1 + \ldots + \gamma_1^{t-1}\right) + \gamma_1^t y_{i0} + \sum_{j=0}^{t-1}\gamma_1^j u_{i,t-j}$$

Or

$$y_{it} = \gamma_0\sum_{j=0}^{t-1}\gamma_1^j + \alpha_i\sum_{j=0}^{t-1}\gamma_1^j + \gamma_1^t y_{i0} + \sum_{j=0}^{t-1}\gamma_1^j u_{i,t-j}$$

Therefore

$$y_{it-1} = \gamma_0\sum_{j=0}^{t-2}\gamma_1^j + \alpha_i\sum_{j=0}^{t-2}\gamma_1^j + \gamma_1^{t-1} y_{i0} + \sum_{j=0}^{t-2}\gamma_1^j u_{i,t-1-j}$$

$For\ large\ t,$

$$E(y_{it}/\alpha_i) = \gamma_0 \frac{1}{1-\gamma_1} + \alpha_i \frac{1}{1-\gamma_1}$$

$$V(y_{it}/\alpha_i) = \frac{\sigma_\varepsilon^2}{1-\gamma_1^2}$$

5.2 Fixed and Random Effects Estimation

$$y_{it} = \gamma_0 + \gamma_1 y_{it-1} + \alpha_i + u_{it} \quad (5.3)$$

Remark: *One possible cause for biasedness is the presence of the unknown individual effects α_i, which creates a correlation between the explanatory variables and the residuals*

$$\left(y_{it} - \overline{y}_i\right) = \gamma_1\left(y_{it-1} - \overline{y}_{i,-1}\right) + u_{it} - \overline{u}_i$$

Notes: $\left(y_{it-1} - \overline{y}_{i,-1}\right)$ *will be correlated* $\left(u_{it} - \overline{u}_i\right)$

$$\left(y_{it}-\bar{y}_i\right)=\gamma_1\left(y_{it-1}-\underbrace{\bar{y}_{i,-1}}_{\text{depen on past value of } u_{it}}\right)+u_{it}-\underbrace{\bar{u}_i}_{\text{depen on past value of } u_{it}}$$

The within estimator or fix effects estimator is

$$\hat{\gamma}_{1FE}=\frac{\sum_{i=1}^{N}\sum_{t=1}^{T}\left(y_{it}-\bar{y}_i\right)\left(y_{it-1}-\bar{y}_{i,-1}\right)}{\sum_{i=1}^{N}\sum_{t=1}^{T}\left(y_{it-1}-\bar{y}_{i,-1}\right)^2}$$

$$=\gamma_1+\frac{\sum_{i=1}^{N}\sum_{t=1}^{T}\left(y_{it-1}-\bar{y}_{i,-1}\right)\left(u_{it}-\bar{u}_i\right)}{\sum_{i=1}^{N}\sum_{t=1}^{T}\left(y_{it-1}-\bar{y}_{i,-1}\right)^2}$$

$$\hat{\alpha}_i=y_i-\hat{\gamma}_{1FE}\,\bar{y}_{i,-1}$$

Problem: Fixed effects the within transformation and LSDV produce biased estimates

$$\hat{\gamma}_{1FE} = \gamma_1 + \frac{\sum_{i=1}^{N}\sum_{t=1}^{T}\left(y_{it-1} - \bar{y}_{i,-1}\right)\left(u_{it} - \bar{u}_i\right) / NT}{\sum_{i=1}^{N}\sum_{t=1}^{T}\left(y_{it-1} - \bar{y}_{i,-1}\right)^2 / NT}$$

Theorem. (Weak law of large numbers, Khinchine)

If $\{X_i\}$ for i=1,…, m is a sequence of i.i.d random variables with $E(X_i) = \mu < \infty$, then the sample mean coverges in probability to μ:

$$\frac{1}{m}\sum_{i=1}^{m} X_i \xrightarrow{p} E(X_i) = \mu \Leftrightarrow p\lim_{m\to+\infty}\frac{1}{m}\sum_{i=1}^{m} X_i = E(X_i) = \mu$$

We have

$$\underset{N\to+\infty}{p\lim}\frac{1}{NT}\sum_{i=1}^{N}\sum_{t=1}^{T}\left(y_{it-1}-\overline{y}_{i,-1}\right)\left(u_{it}-\overline{u}_{i}\right)$$

$$=\underbrace{\underset{N\to+\infty}{p\lim}\frac{1}{NT}\sum_{i=1}^{N}\sum_{t=1}^{T}y_{it-1}u_{it}}_{N_1}-\underbrace{\underset{N\to+\infty}{p\lim}\frac{1}{NT}\sum_{i=1}^{N}\sum_{t=1}^{T}y_{it-1}\overline{u}_{i}}_{N_2}$$

$$-\underbrace{\underset{N\to+\infty}{p\lim}\frac{1}{NT}\sum_{i=1}^{N}\sum_{t=1}^{T}\overline{y}_{i,-1}u_{it}}_{N_3}+\underbrace{\underset{N\to+\infty}{p\lim}\frac{1}{NT}\sum_{i=1}^{N}\sum_{t=1}^{T}\overline{y}_{i,-1}\overline{u}_{i}}_{N_4}$$

$$N_1=\underset{N\to+\infty}{p\lim}\frac{1}{NT}\sum_{i=1}^{N}\sum_{t=1}^{T}y_{it-1}u_{it}=E\left(y_{it-1}u_{it}\right)=0$$

$$N_2=\underset{N\to+\infty}{p\lim}\frac{1}{NT}\sum_{i=1}^{N}\sum_{t=1}^{T}y_{it-1}\overline{u}_{i}=\underset{N\to+\infty}{p\lim}\frac{1}{NT}\sum_{i=1}^{N}\overline{u}_{i}\sum_{t=1}^{T}y_{it-1}$$

$$=\underset{N\to+\infty}{p\lim}\frac{1}{NT}\sum_{i=1}^{N}\overline{u}_{i}T\overline{y}_{i,-1}=\underset{N\to+\infty}{p\lim}\frac{1}{N}\sum_{i=1}^{N}\overline{u}_{i}\overline{y}_{i,-1}$$

$$N_3 = \underset{N\to+\infty}{p\lim}\frac{1}{NT}\sum_{i=1}^{N}\sum_{t=1}^{T}\overline{y}_{i,-1}u_{it} = \underset{N\to+\infty}{p\lim}\frac{1}{NT}\sum_{i=1}^{N}\overline{y}_{i,-1}\sum_{t=1}^{T}u_{it} = \underset{N\to+\infty}{p\lim}\frac{1}{N}\sum_{i=1}^{N}\overline{y}_{i,-1}\overline{u}_i$$

$$N_4 = \underset{N\to+\infty}{p\lim}\frac{1}{NT}\sum_{i=1}^{N}\sum_{t=1}^{T}\overline{y}_{i,-1}\overline{u}_i = \underset{N\to+\infty}{p\lim}\frac{1}{NT}T\sum_{i=1}^{N}\overline{y}_{i,-1}\overline{u}_i = \underset{N\to+\infty}{p\lim}\frac{1}{N}\sum_{i=1}^{N}\overline{y}_{i,-1}\overline{u}_i$$

$$\underset{N\to+\infty}{p\lim}\frac{1}{NT}\sum_{i=1}^{N}\sum_{t=1}^{T}\left(y_{it-1}-\overline{y}_{i,-1}\right)\left(u_{it}-\overline{u}_i\right) = -\underset{N\to+\infty}{p\lim}\frac{1}{N}\sum_{i=1}^{N}\overline{u}_i\,\overline{y}_{i,-1}$$

$$0-\underset{N\to+\infty}{p\lim}\frac{1}{N}\sum_{i=1}^{N}\overline{u}_i\,\overline{y}_{i,-1}-\underset{N\to+\infty}{p\lim}\frac{1}{N}\sum_{i=1}^{N}\overline{u}_i\,\overline{y}_{i,-1}+\underset{N\to+\infty}{p\lim}\frac{1}{N}\sum_{i=1}^{N}\overline{u}_i\,\overline{y}_{i,-1}$$

$$\hat{\gamma}_{1FE} = \gamma_1 + \frac{\sum_{i=1}^{N}\sum_{t=1}^{T}\left(y_{it-1} - \overline{y}_{i,-1}\right)\left(u_{it} - \overline{u}_i\right)/NT}{\sum_{i=1}^{N}\sum_{t=1}^{T}\left(y_{it-1} - \overline{y}_{i,-1}\right)^2/NT} = \gamma_1 - \underset{N\to+\infty}{p\lim}\frac{1}{N}\sum_{i=1}^{N}\overline{u}_i\,\overline{y}_{i,-1}$$

If this plim is not null, then the $\hat{\gamma}_{1,FE}$ estimator is biased when N tends to infinity and T is fixed

Fact. If T also tends to infinity, then the numerator converges to zero

Pre Example (3.1) With model

ROAA = f(L.ROAA, HHI, L_A, SIZE, ASSET_GRO, GDP, INF) + ε

5.3 Intrustment Variable Estiamtor

The problem of estimation involved in the fixed effects dynamic model can also be looked into by expressing Eq. (5.3) in vector form:

$$y_i = \gamma_0 e + \gamma_1 y_{i,-1} + \alpha_i e + u_i \qquad (5.4)$$

Here

$$y_i = \begin{pmatrix} y_{i1} \\ y_{i2} \\ \cdots \\ y_{iT} \end{pmatrix}_{T\times 1} ; e = \begin{pmatrix} 1 \\ 1 \\ \cdots \\ 1 \end{pmatrix}_{T\times 1} ; y_{i,-1} = \begin{pmatrix} y_{i0} \\ y_{i,-1} \\ \cdots \\ y_{iT-1} \end{pmatrix}_{T\times 1} ; u_i = \begin{pmatrix} u_{i1} \\ u_{i2} \\ \cdots \\ u_{iT} \end{pmatrix}_{T\times 1}$$

Pre-multiplying both sides of (5.4) by Q, the model is transformed as

$$Qy_i = \gamma_0 Qe + \gamma_1 Qy_{i,-1} + \alpha_i Qe + Qu_i$$

$$\hat{\gamma}_{1,FE} = \sum_{i=1}^{N}\left(y_{i,-1}^{'}Qy_{i,-1}\right)\sum_{t=1}^{N} y_{i,-1}^{'}Qy_{i}$$

Or

$$\hat{\gamma}_{1,FE} = \gamma_1 + \sum_{i=1}^{N}\left(y_{i,-1}^{'}Qy_{i,-1}\right)\sum_{t=1}^{N} y_{i,-1}^{'}Qu_{i}$$

Now, $\hat{\gamma}_{1,FE}$ will be unbiased and consistent when

$$\underset{N\to+\infty}{p\lim}\frac{1}{N}\sum_{t=1}^{N} y_{i,-1}^{'}Qu_{i} = E\left(y_{i,-1}^{'}Qy_{i,-1}\right) = 0$$

But

$$E\left(y_{i,-1}^{'}Qy_{i,-1}\right) = E\left(y_{i0} \quad y_{i,-1} \quad \dots \quad y_{i,T-1}\right)\begin{pmatrix} u_{i1} - \bar{u}_i \\ u_{i2} - \bar{u}_i \\ \dots \\ u_{iT} - \bar{u}_i \end{pmatrix}$$

$$= E\left(\sum_{t=1}^{T} y_{i,t-1}\left(u_{it} - \frac{1}{T}\sum_{t=1}^{T} u_{it}\right)\right) \neq 0$$

This is because

$$E\left(y_{i,t-1}\left(u_{it}-T^{-1}\sum_{t=1}^{T}u_{it}\right)\right)E=\left(\sum_{j=0}^{T-2}\gamma_1^j u_{i,t-1-j}\left(u_{it}-T^{-1}\sum_{t=1}^{T}u_{it}\right)\right)\neq 0$$

5.1 Introduction

Linear dynamic panel data models include lag dependent variables as covariates along with the unobserved effects, fixed or random, and exogenous regressors

We consider a dynamic panel model, in the sense that it contains (at least) one lagged variables. For simplicity, let us consider

$$y_{it} = \gamma y_{it-1} + \alpha_0 + x_{it}\beta + \alpha_i + u_{it} = \gamma y_{it-1} + x_{it}\beta + \alpha_i^* + u_{it}$$

$$y_{it} = \gamma y_{it-1} + \alpha_0 + \alpha_i + \beta'_{it}x_{it} + u_{it}$$

5.2 Introduce the autoregressive (AR) panel data model

Or $\Delta y_{it} = \alpha_i^* + \rho y_{it-1} + u_{it}$ (5.2)

Consider the simple the AR(1) model

Here $\rho = \gamma - 1$

$$y_{it} = \gamma y_{it-1} + \alpha_i^* + u_{it} \qquad (5.1)$$

5.3 Panel Unit Root Test

hh